For a rigid wall $\varrho_0' D_2 \to \infty$ and eq. (37) becomes

$$\frac{p_2 - p_0}{p_1 - p_0} = \frac{\varrho_1 D_{21}' + \varrho_0 D_1}{\varrho_0 D_1} \xrightarrow[p_1 \to p_0]{} 2 . \tag{39}$$

For a free surface, $\varrho_0' D_2 = 0$ and eq. (37) becomes

$$(p_2 - p_0)/(p_1 - p_0) = 0 . \tag{40}$$

Equation (37) applies exactly only when the reflected wave is a shock. Because of the second-order contact between the rarefaction branch and the shock branch of the cross curve passing through A, the formula provides a very good approximation for rarefactions in condensed materials.

Figure 11: Weak-shock approximation: Equations (32) and (35) yield very nearly the same curve when $(V_0 - V)/V_0 \lesssim 0.15$. Then either can be used and the (p, u) plane can be mapped with a set of identical curves and their mirror images. These curves can be regarded as transformations from the (x, t) plane. A curve along which transitions can be made by forward-facing waves is an image of a $C-$ characteristic and is called a Γ_- characteristic.

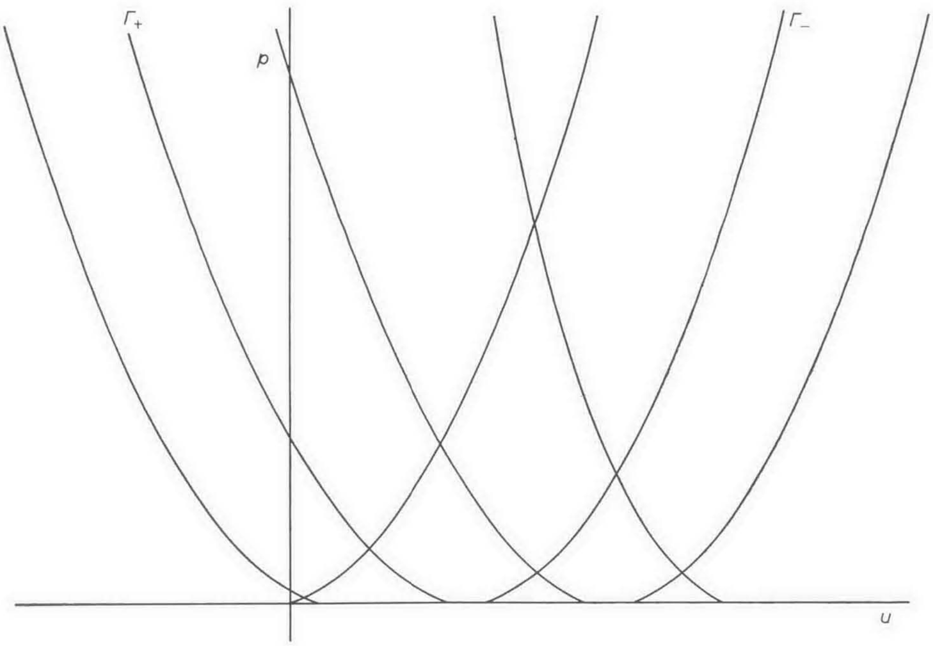

Fig. 11. – Weak shock approximation, $S = \text{const}$. No distinction between adiabat and Hugoniot; $(p\text{-}u)$ plane can be mapped with images of the $C+$ and $C-$ characteristics. $u - l = \text{const}$ on Γ_-, $u + l = \text{const}$ on Γ_+.

Transitions through backward-facing waves are along Γ_+ characteristics. Transformation from the (x, t) to the (x, u) plane is not necessarily one-one. For example, a region of uniform state in the (x, t) plane maps into a single point in the (p, u) plane; a simple wave in the (x, t) plane maps into one of the Γ characteristics.

An example of application of the characteristic mapping of the (p, u) plane in the weak shock approximation is shown in Fig. 12. A forward-facing rarefaction travels into a uniform state $(p_0, u_0 = 0)$ bounded by a rigid wall, reducing it to the uniform state (p_1, u_1). The reflection process is represented in the (x, t) plane by drawing $C+$ and $C-$ characteristics at arbitrarily close intervals and labelling the fields between the characteristics as shown. In relating this flow to the (p, u) plane, each field in the (x, t) plane is considered a region of uniform state, represented by a single point in the (p, u) plane. Thus the regions 00, 01, 02, 03 map into the points 00, 01, 02, 03 on the Γ_- characteristic $(u - l = l_0)$ passing through $(p_0, u_0 = 0)$. Region 11 in (x, t) is reached from 01 across a backward-facing wave, therefore it lies on a Γ_+ characteristic passing through 01. The rigid boundary condition $u = 0$ must be satisfied in 11, therefore 11 in (p, u) lies at the intersection of the Γ_+ through 01 and the $u = 0$ axis. Similarly the transition from 02 to 12 to 22 takes place along a Γ_+ characteristic and terminates at $u = 0$. The intermediate state 12 is reached via a backward-facing wave from 02 and a forward-facing wave from 11, etc. This simple step by step procedure will succeed in unraveling the most complicated flow problems in (x, t) geometry provided only that the weak shock approximation is valid. With the aid of a large drawing board and considerable patience, the procedure is useful for graphical computation; and the solution of a few problems by this means is certain to establish the elements of finite amplitude wave propagation firmly in the mind of the student. It should be noted that the C characteristics are actually curved in the region of interaction, and the slopes of those characteristics

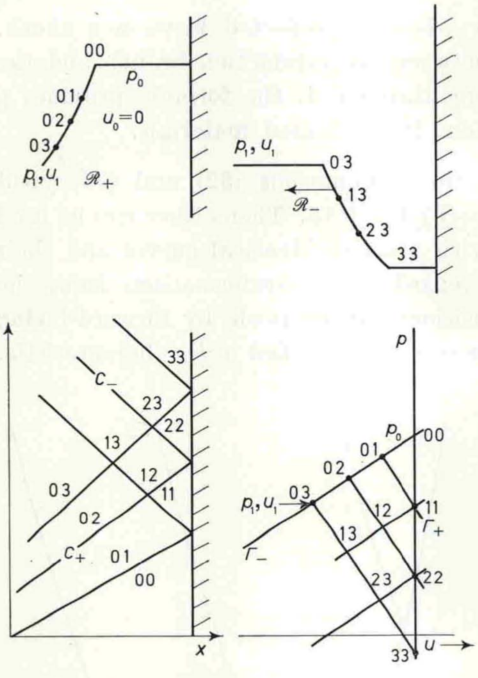

Fig. 12. – Rarefaction on a rigid wall.